Scut: A Balanced Clustering Algorithm

Mikko Malinen

Speech and Image Processing Unit
School of Computing
University of Eastern Finland

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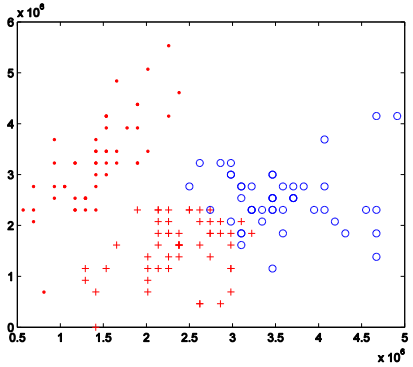
UNIVERSITY OF EASTERN FINLAND

University of
Eastern Finland
School of Computing
P.O. Box 111
FIN- 80101 Joensuu
FINLAND

Tel. +358 2944 53648
fax +358 13 251 7955

www.uef.fi/cs

A more balanced clustering result is desirable in some applications (f.eg. in photo query systems, workload balancing)



$$TSE_i = \sum_{\substack{j,\text{ so that} \\ X_i \text{ belongs} \\ \text{to cluster } i}} | X_j - C_i |^2$$

We present a clustering method, which weights TSE of a single cluster by number of points in cluster and uses those weighted TSE's as a cost.

$$\text{Cost} = n_1 \cdot TSE_1 + n_2 \cdot TSE_2 + \ldots + n_k \cdot TSE_k,$$

where n_i is number of points in ith cluster.

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The cost function

$$\text{Cost} = n_1 \cdot TSE_1 + n_2 \cdot TSE_2 + \ldots + n_k \cdot TSE_k,$$

where n_i is number of points in ith cluster.

may also be written as

$$\text{Cost} = {n_1}^2 \cdot MSE_1 + {n_2}^2 \cdot MSE_2 + \ldots + {n_k}^2 \cdot MSE_k,$$

where n_i is number of points in ith cluster.

$$(MSE_i = \sum_{\substack{j, \text{so that} \\ X_j \text{ belongs} \\ \text{to cluster } i}} \frac{|X_j - C_i|^2}{n_i})$$

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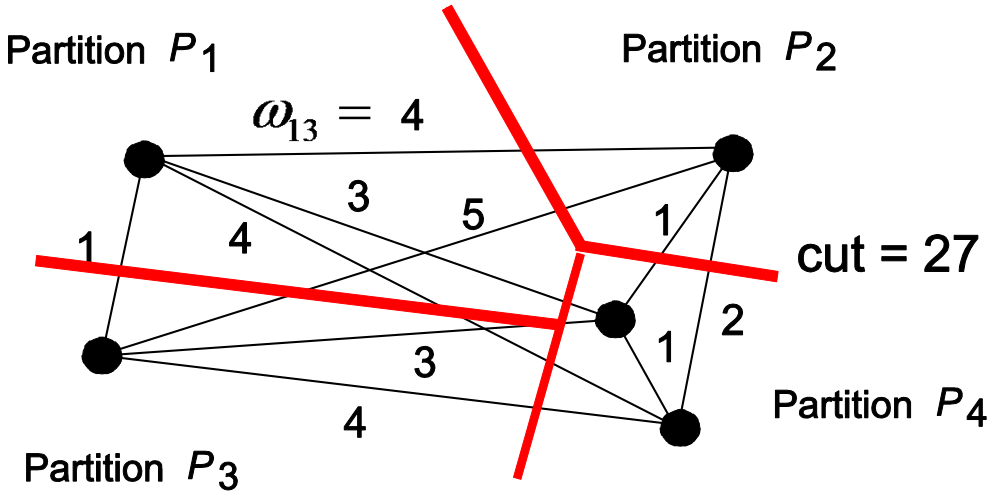
MAX k-CUT



MAX 4-CUT

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Scut algorithm

Input: dataset X, number of clusters k

Output: partitioning of points P

Set the vertices of the graph the data points.

Set the weights of the edges of the graph the squared distances.

Compute MAX k-CUT

Output partitioning of points P

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Balanced k-means clustering

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Balanced clustering

Want to fix the number of points in clusters?

We have a solution for that.

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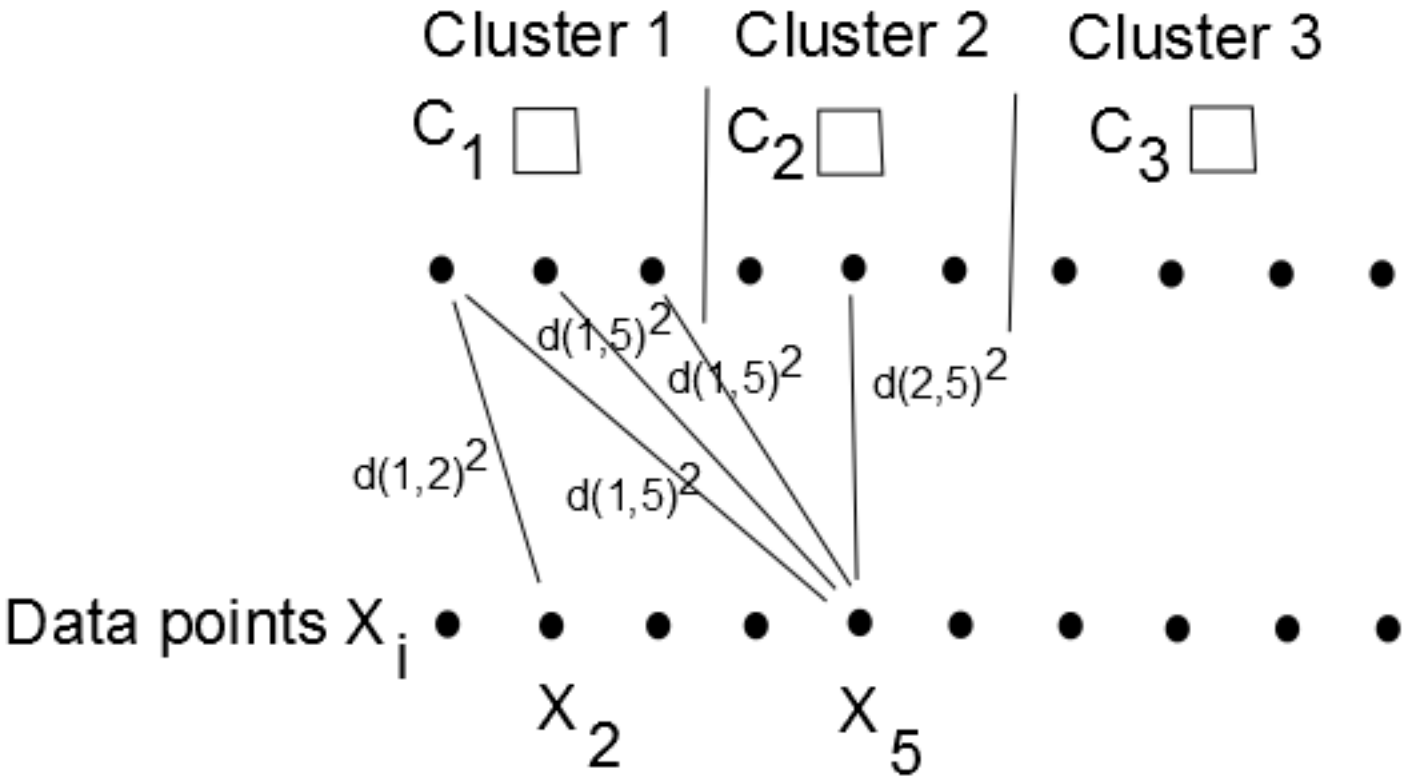
Forming the assignment graph



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K-means –like process

1) **Initialization** of centroid locations
2) **Assignment phase**. Calculate arclengths and use Hungarian algorithm to assign.
3) **Update phase**. Calculate new centroid locations.

Repeat phases 2)-3) until centroid locations do not change.

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Time complexity

Balanced k-means:

$$T(n) = O(I \cdot n^3)$$

Constrained k-means (from literature):

$$T(n) = O(I \cdot k^{3.5} n^{3.5})$$

Where *I* = number of kmeans iterations

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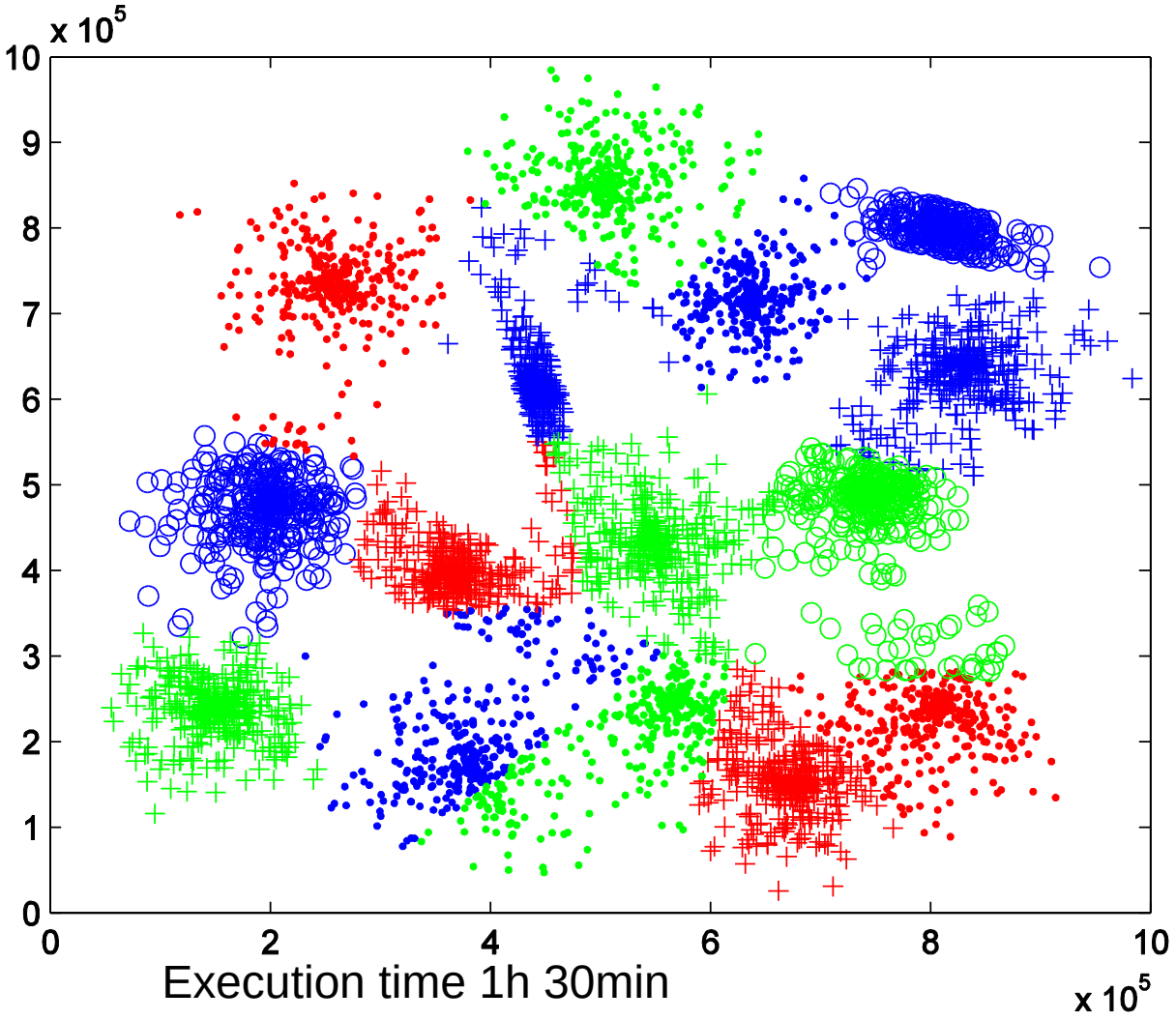
Experiment with dataset s2 of size n=5000



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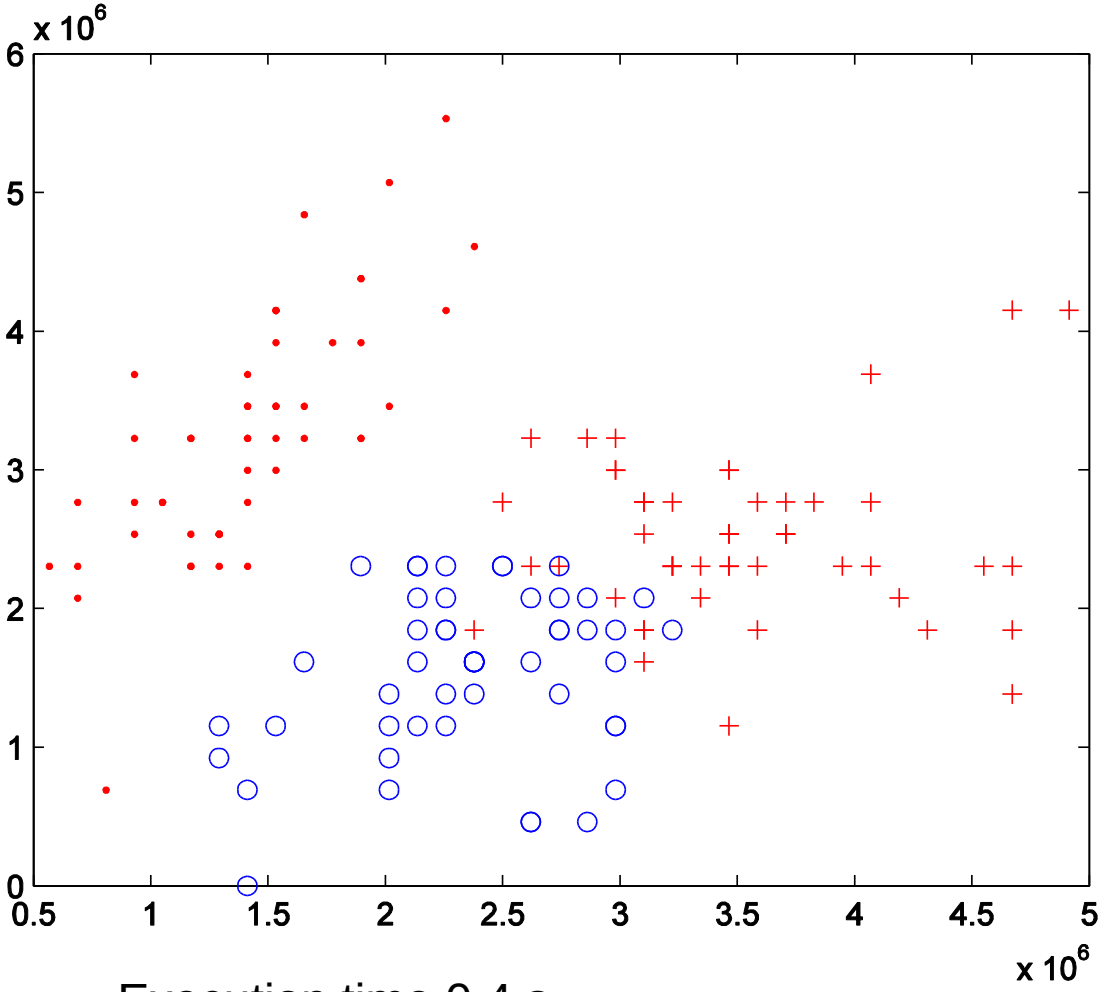
Experiment with Iris dataset of 150 points and 4 dimensions



Execution time 0.4 s

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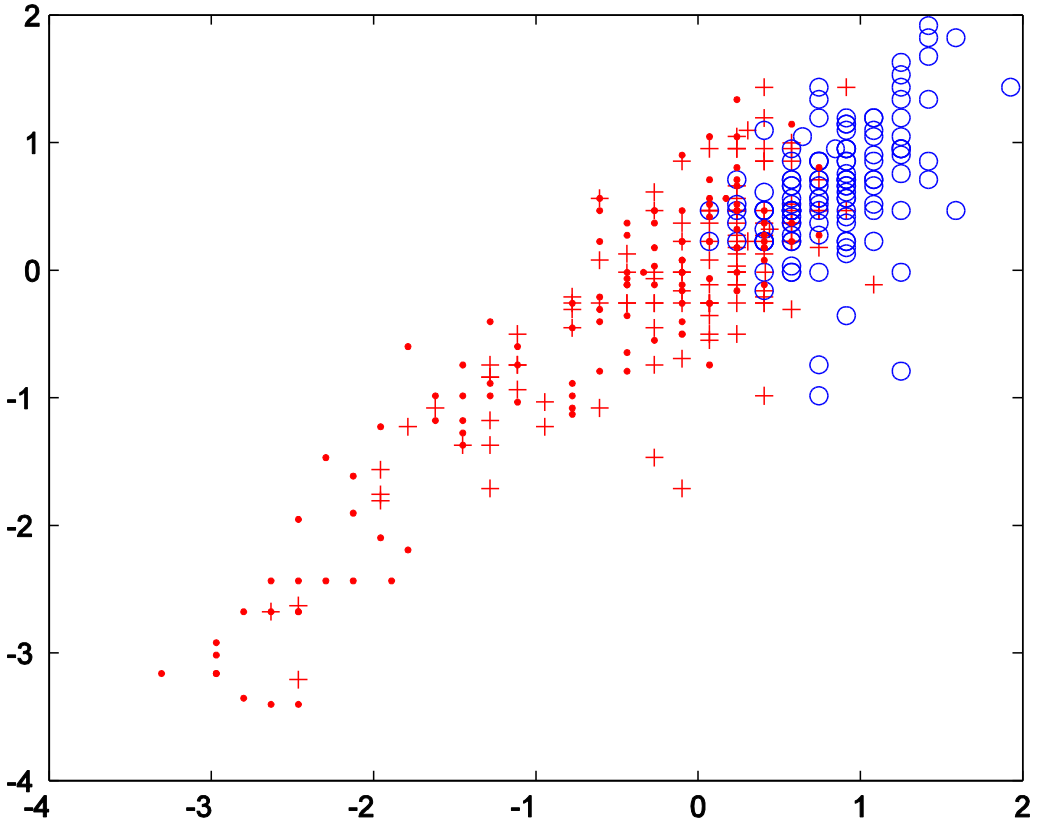
Rottavalikoitu data, normalized, dimensions 2 and 3 out of 14



Execution time 2.6 seconds

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