

Tiedettä Tiedosta (Ja Tiedon Käsittelystä)



Joensuun Yliopisto

Tietojenkäsittelytieteen ja tilastotieteen laitos







Theorem 1. Let $f(x)$ be a function defined on the interval $[a, b]$. Then the remainder $R_n(x)$ of the Taylor series expansion of $f(x)$ about x_0 is given by

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} (x-x_0)^{n+1},$$

where ξ is some number between x_0 and x .

Proof. We start with the Taylor polynomial of degree n :

$$T_n(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n.$$

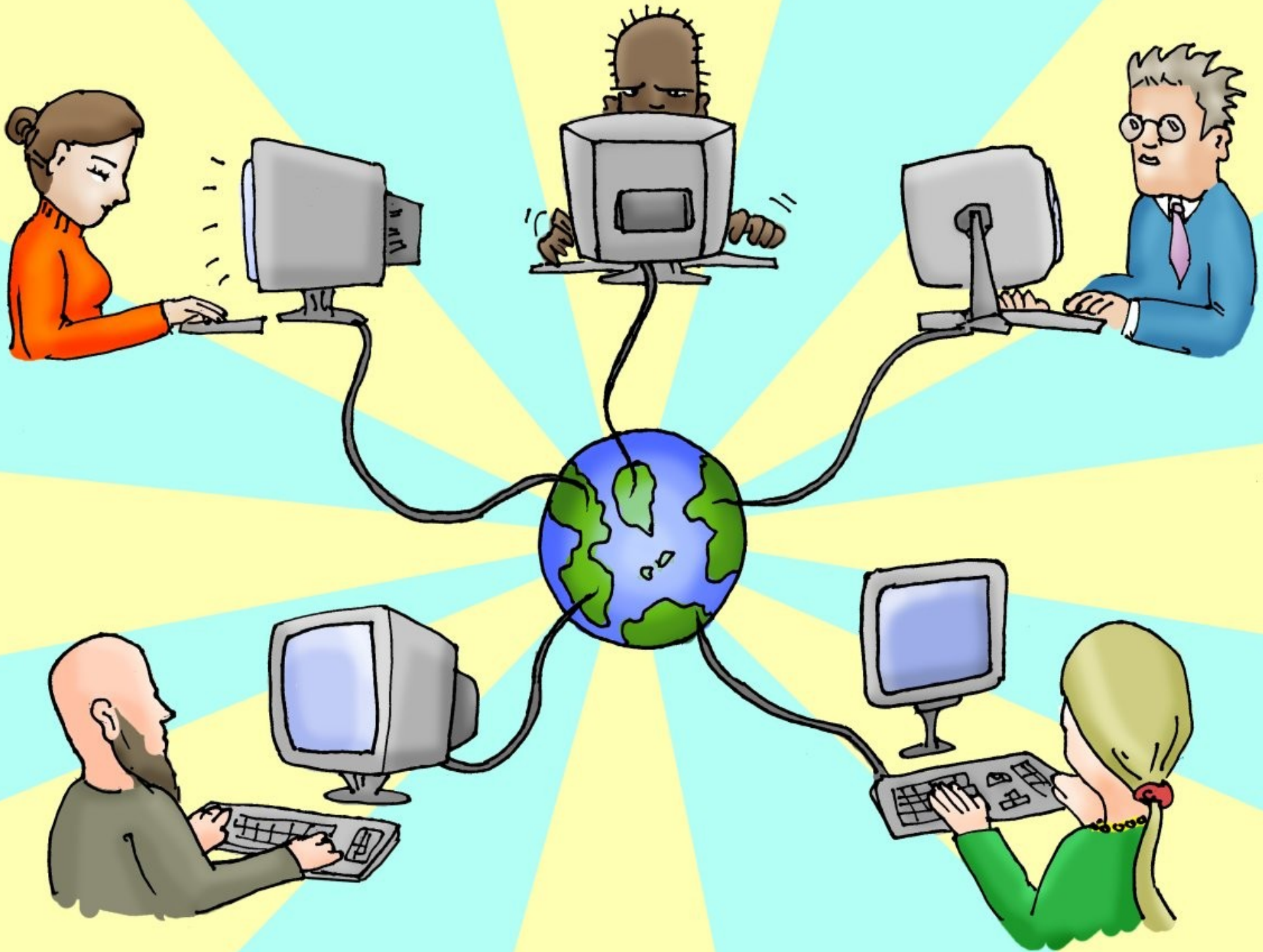
Let $R_n(x) = f(x) - T_n(x)$. Then

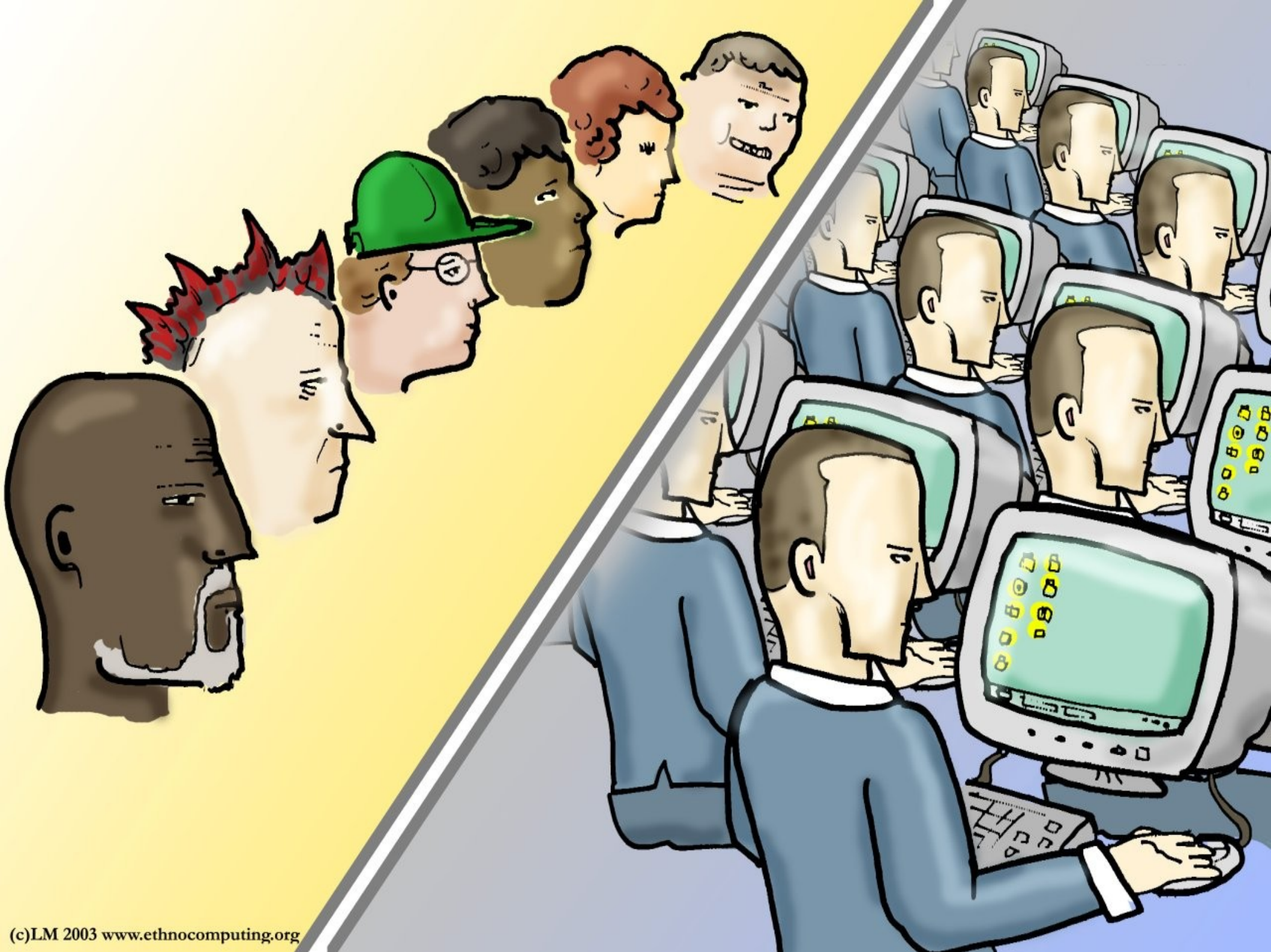
$$R_n(x) = \int_{x_0}^x \frac{f^{(n+1)}(t)}{n!} (x-t)^n dt.$$

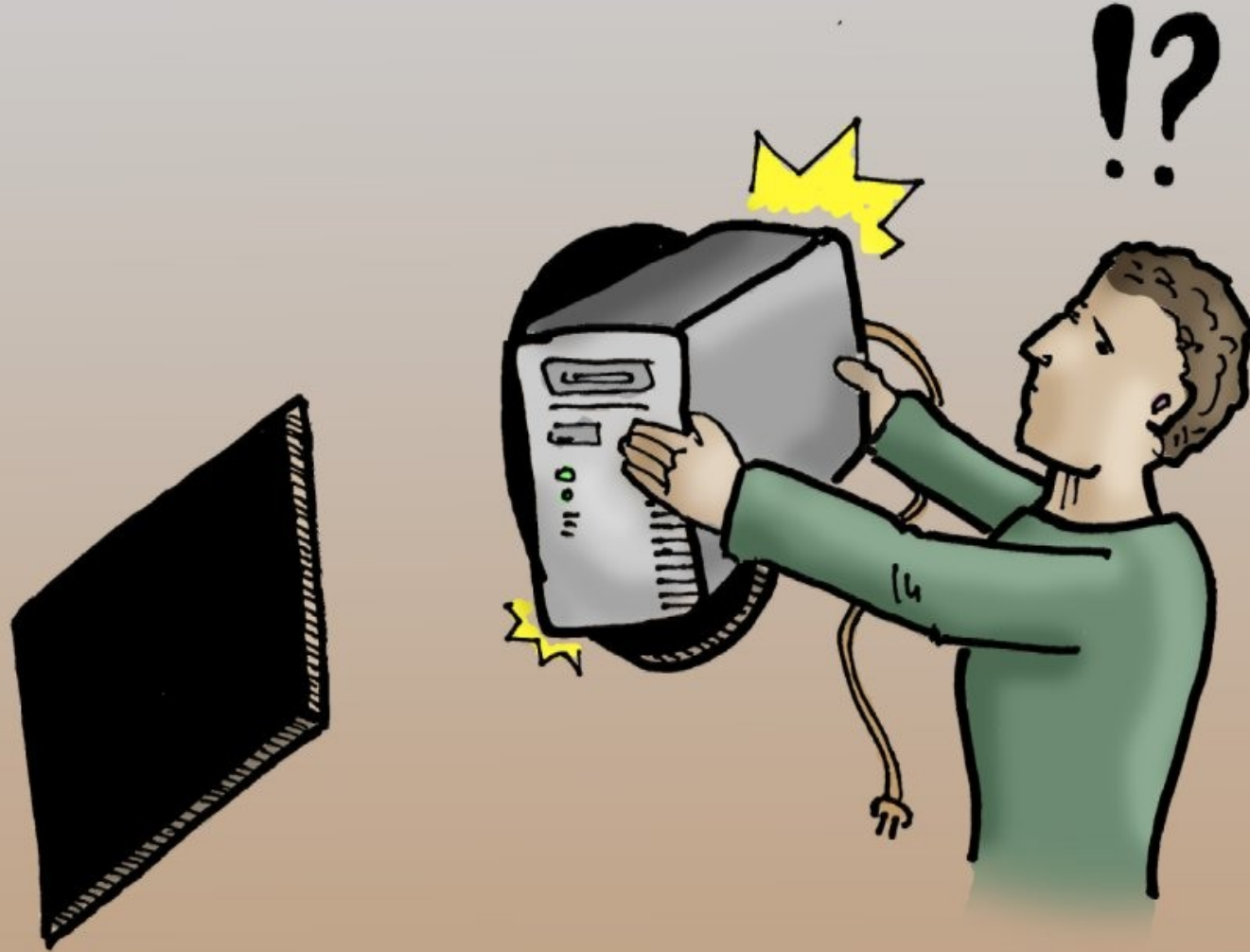
By the Mean Value Theorem for Integrals, there exists a ξ between x_0 and x such that

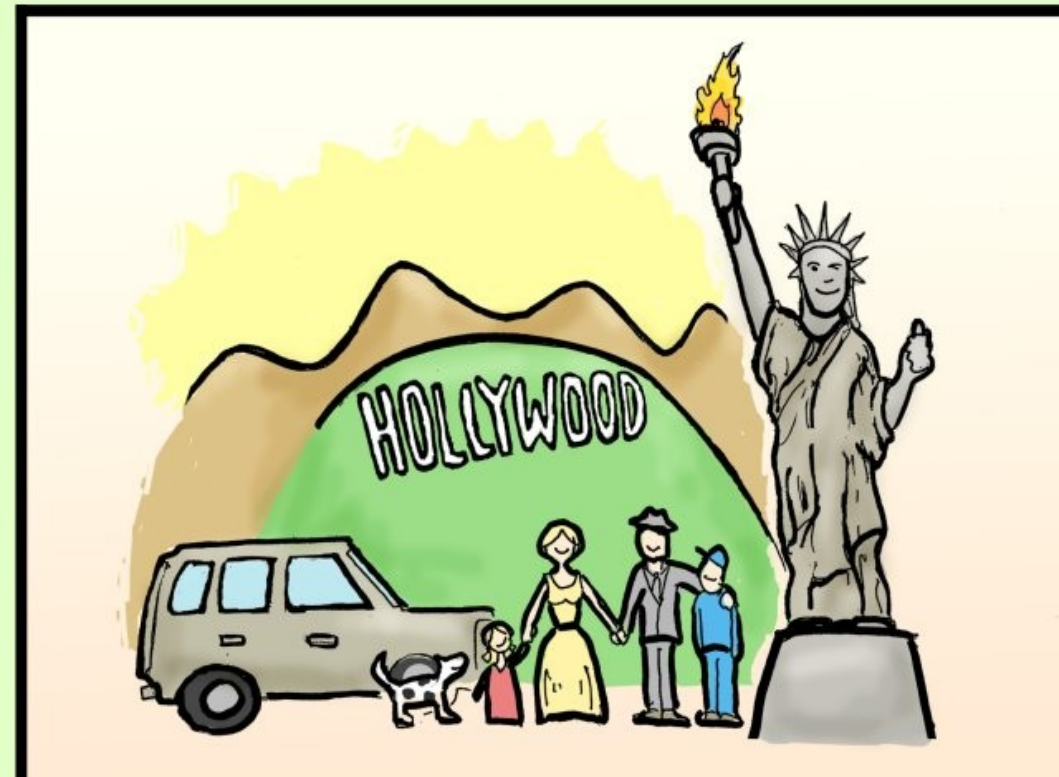
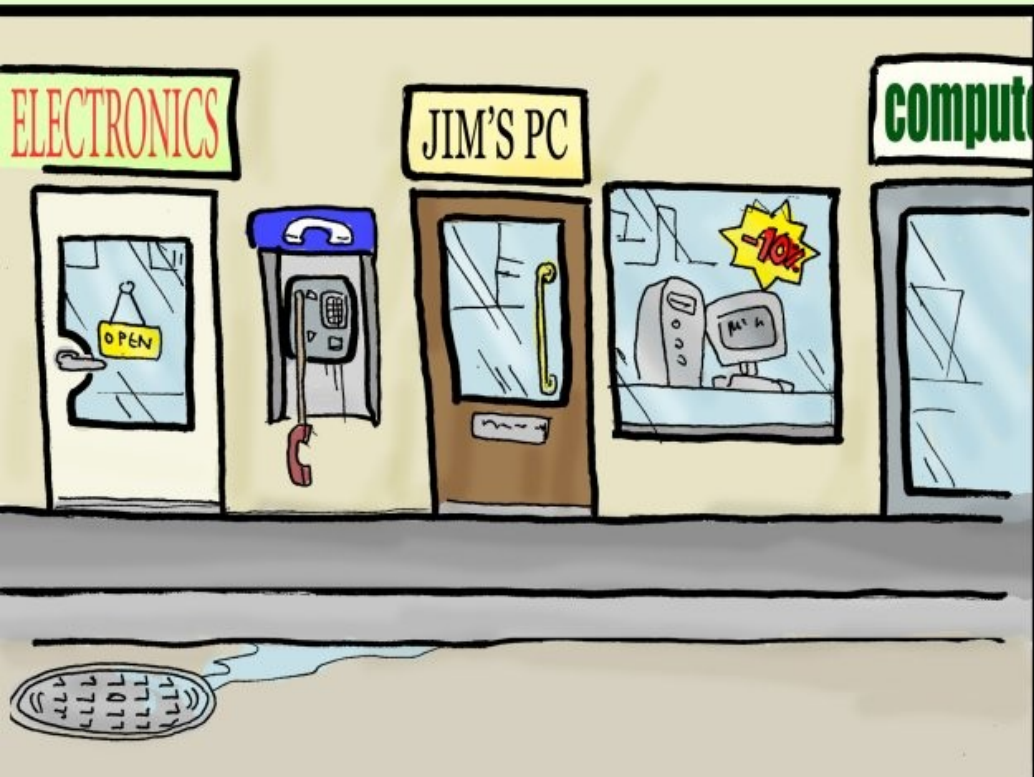
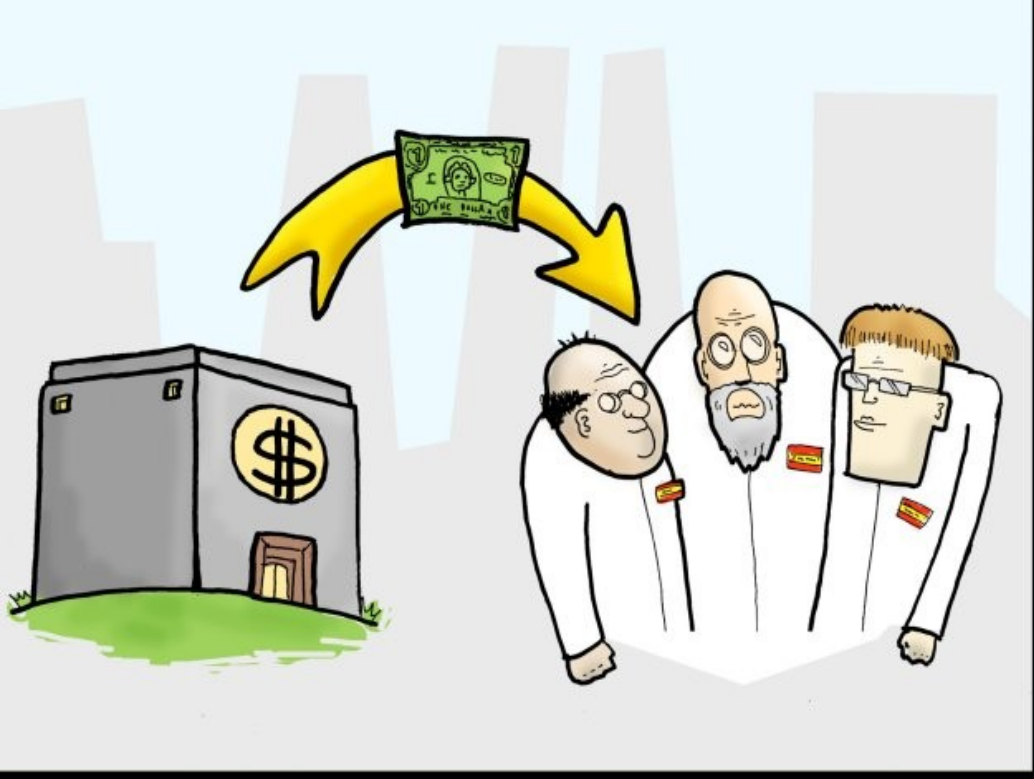
$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} (x-x_0)^{n+1}.$$

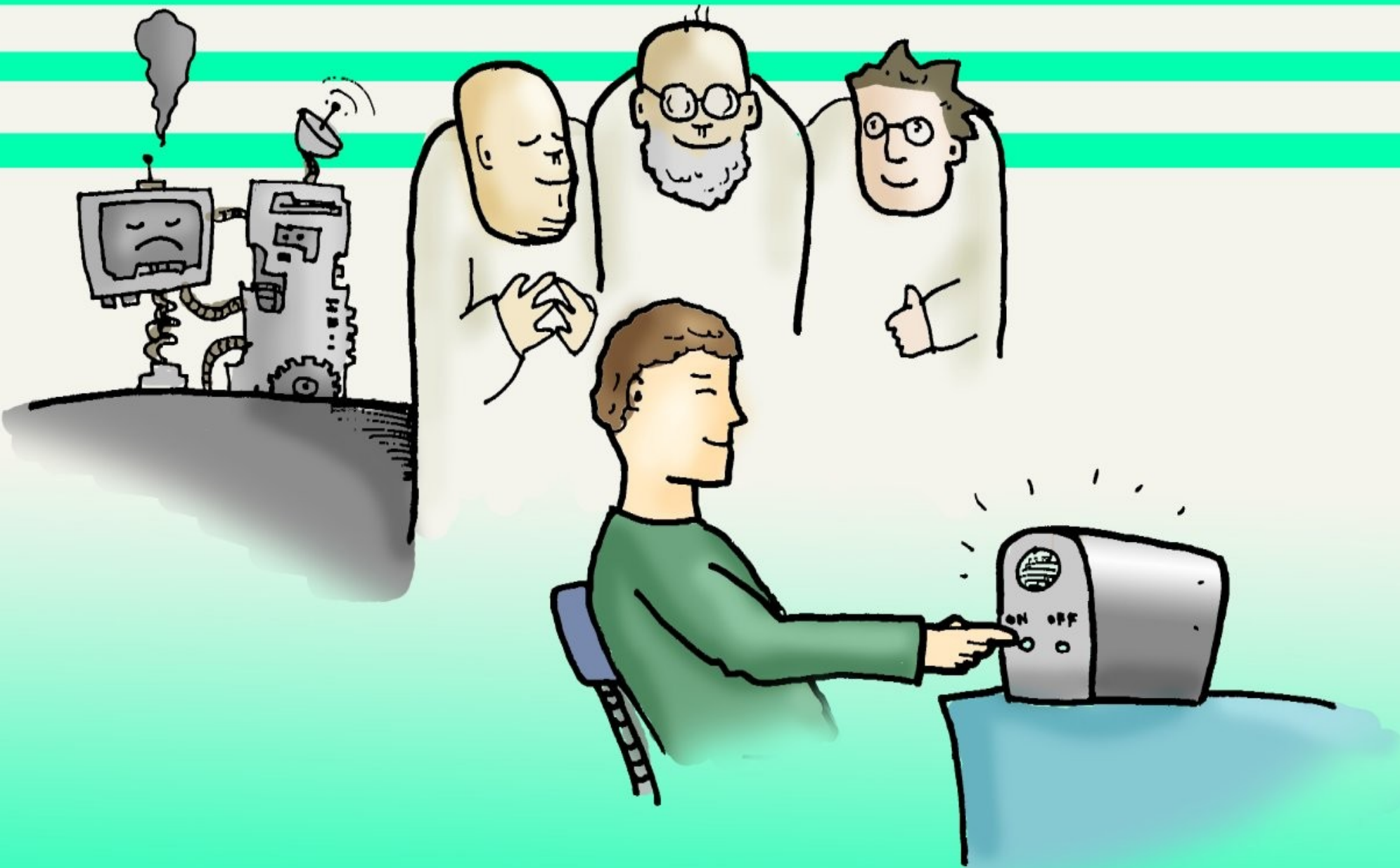


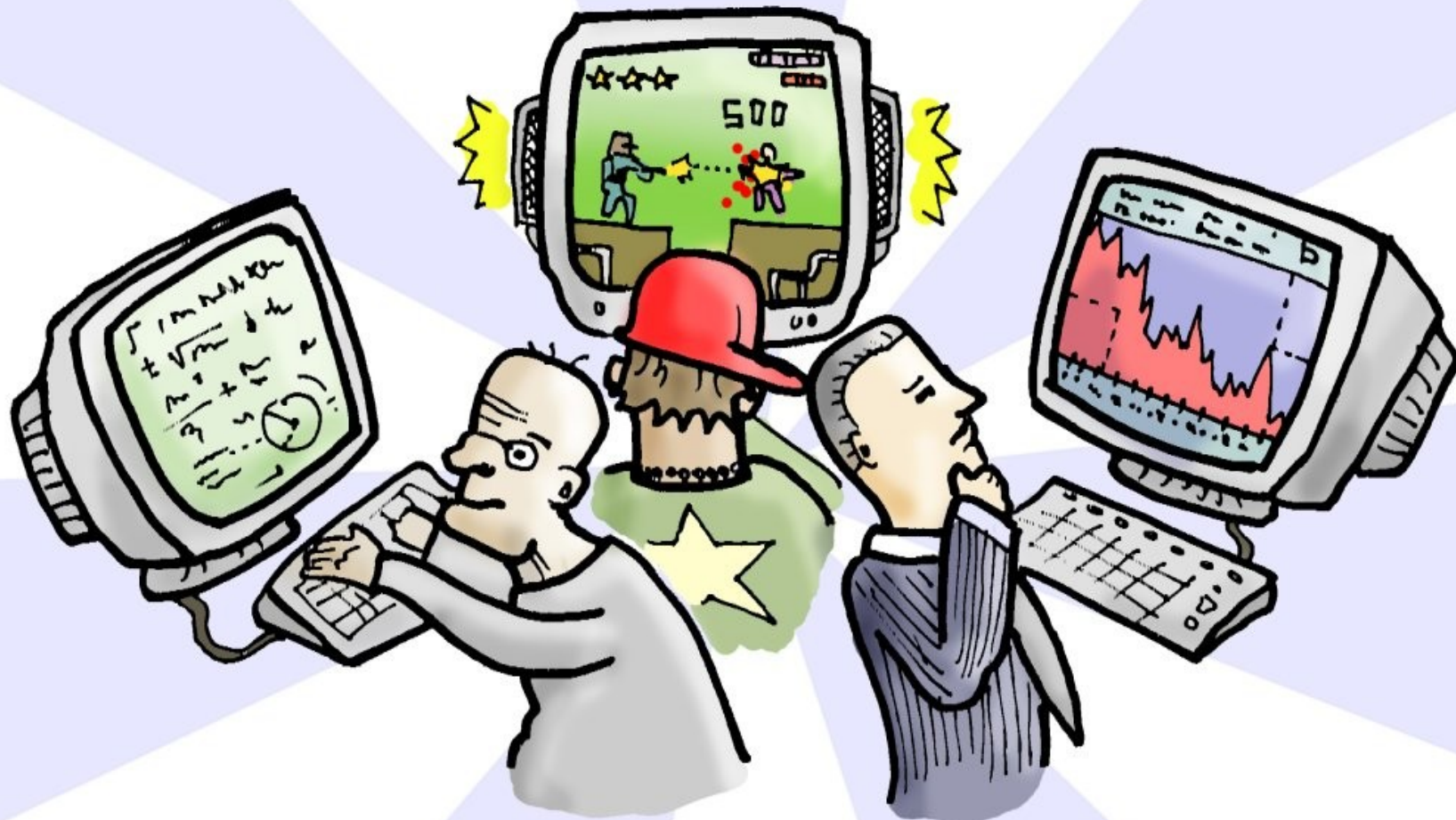

















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