Non-negative Matrix Factorization
NMF

Non-negative Tensor Factorization
NTF

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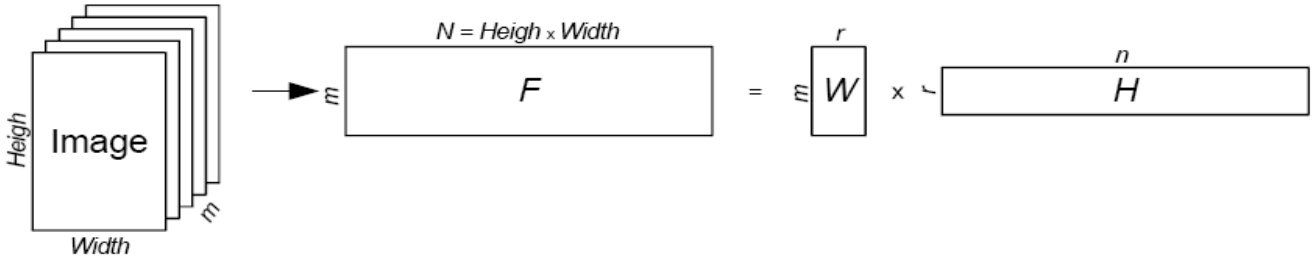
Non-negative Matrix Factorization (NMF)

Given a data matrix $F = \{Fij\}mxn$, non-negative matrix factorization refers to the decomposition of the matrix F into matrices W and H of size nxr and rxm, respectively, such that:

$$F = WH$$

Where the elements in W and H are all positive values. From this decomposition, a reduced representation is achieved by choosing r such that $r<n$ and $r<m$.



NMF

$$\min_{S_o, S_r \geq 0} \| S_o - WH \|$$

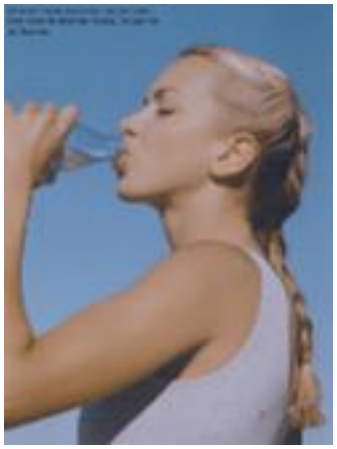
- where *So* is the original spectrum, and *W* and *H* are 2D non-negative matrixes.
- NMF the following iterative learning rules are used to find the linear decomposition:

$$H_{a\mu} \leftarrow H_{a\mu} \sum_i H_{a\mu} \frac{V_{i\mu}}{(WH)_{i\mu}}$$

$$W_{ia} \leftarrow W_{ia} \sum_\mu \frac{V_{i\mu}}{(WH)_{i\mu}} H_{a\mu} \qquad W_{ia} \leftarrow \frac{W_{ia}}{\sum_j W_{ja}}$$

Results

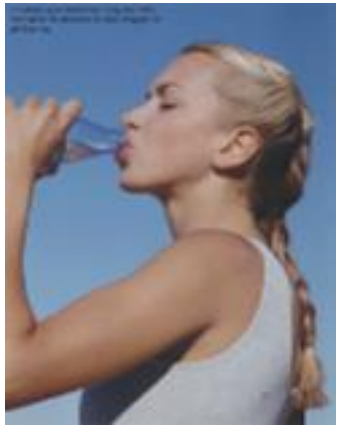
r = 1;



r = 2;

r = 3;

r = 5;



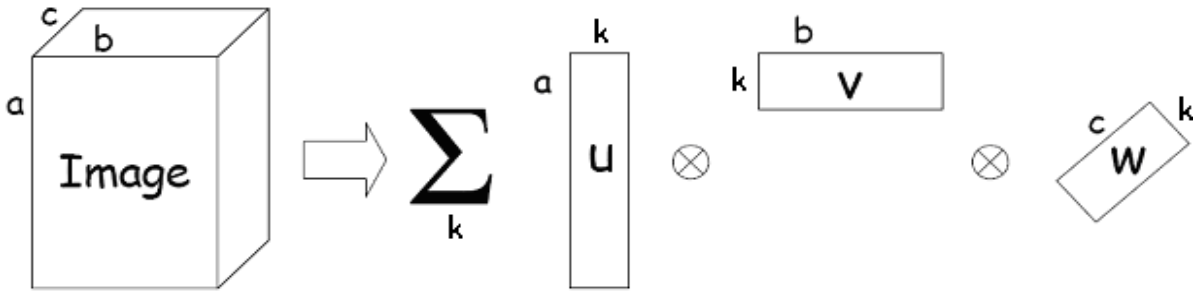
r = 81;

original

Non-negative Tensor Factorization (NTF)

- The basic approach of NTF is to find a solution for the problem:

$$\min_{u^m, v^m, w^m \geq 0} \left\| G - \sum_{m=1}^{k} u^m \otimes v^m \otimes w^m \right\|$$



NTF

Although NTF is originally 3D, one domain can be neglected, since the data is only two-dimensional, the first domain is the spectral domain and the second domain consists of the large number of spectra *n*.

$$\min_{u^m, v^m \geq 0} \left\| S_o - uv^T \right\|$$

The iteration step to update all *i* values is defined as following:

$$u_i^j \leftarrow \frac{u_i^j \sum_{k=1}^{n} S_{i,k} v_k^j}{\sum_{k=1}^{r} u_i^k < v^k, v^j >} \qquad v_i^j \leftarrow \frac{v_i^j \sum_{k=1}^{m} S_{k,i} u_r^j}{\sum_{k=1}^{r} v_i^k < u^k, u^j >}$$

Results

m = 1;

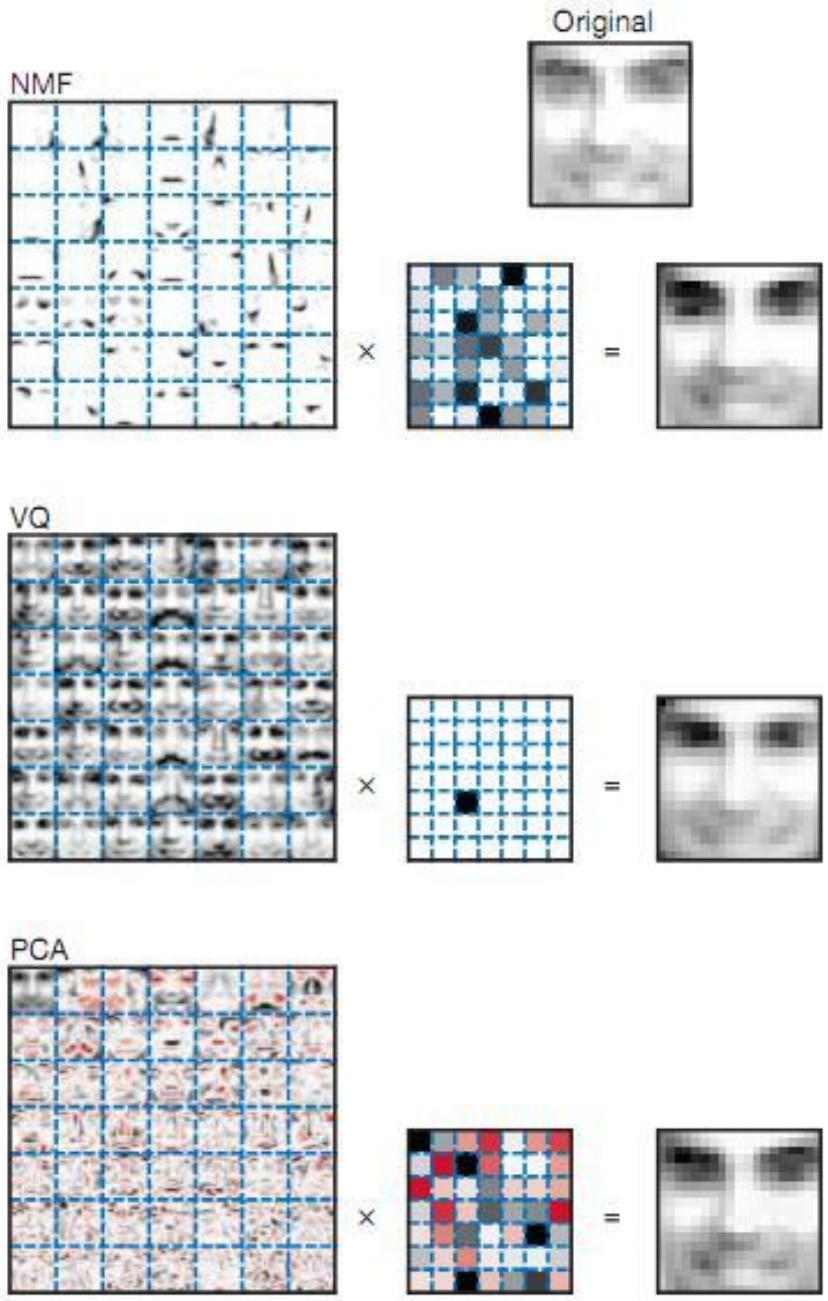
m = 5;

m = 10;

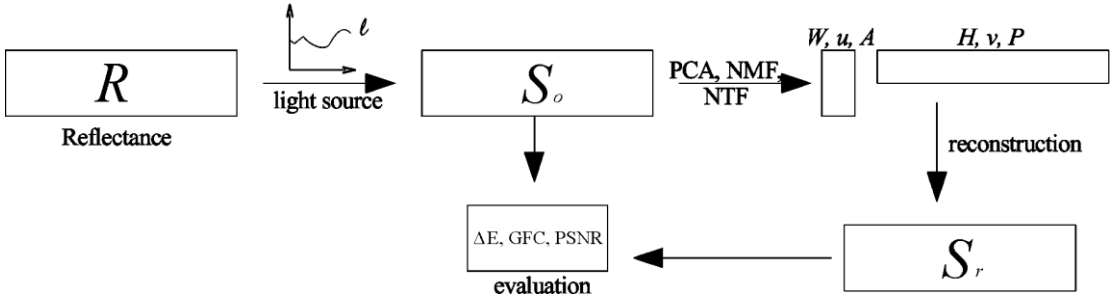
m = 50;

m = 100;

original

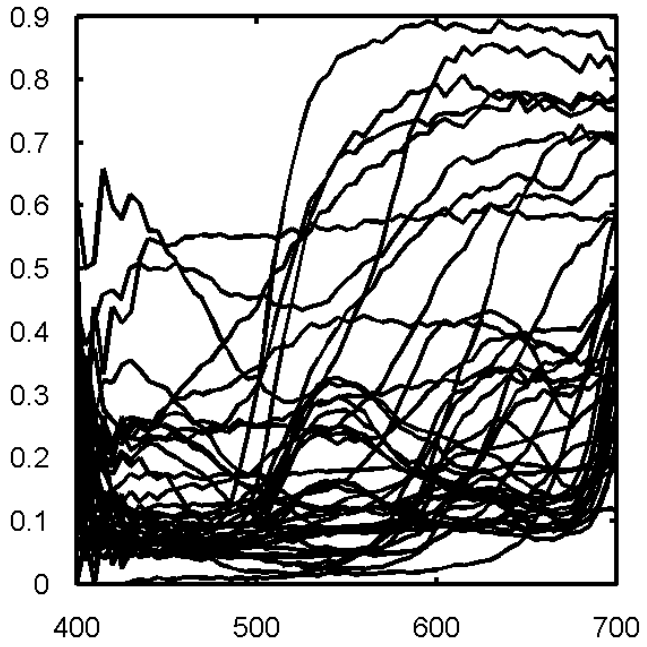

Original
NMF
×
=
VQ
×
=
PCA
×
=

Experiments

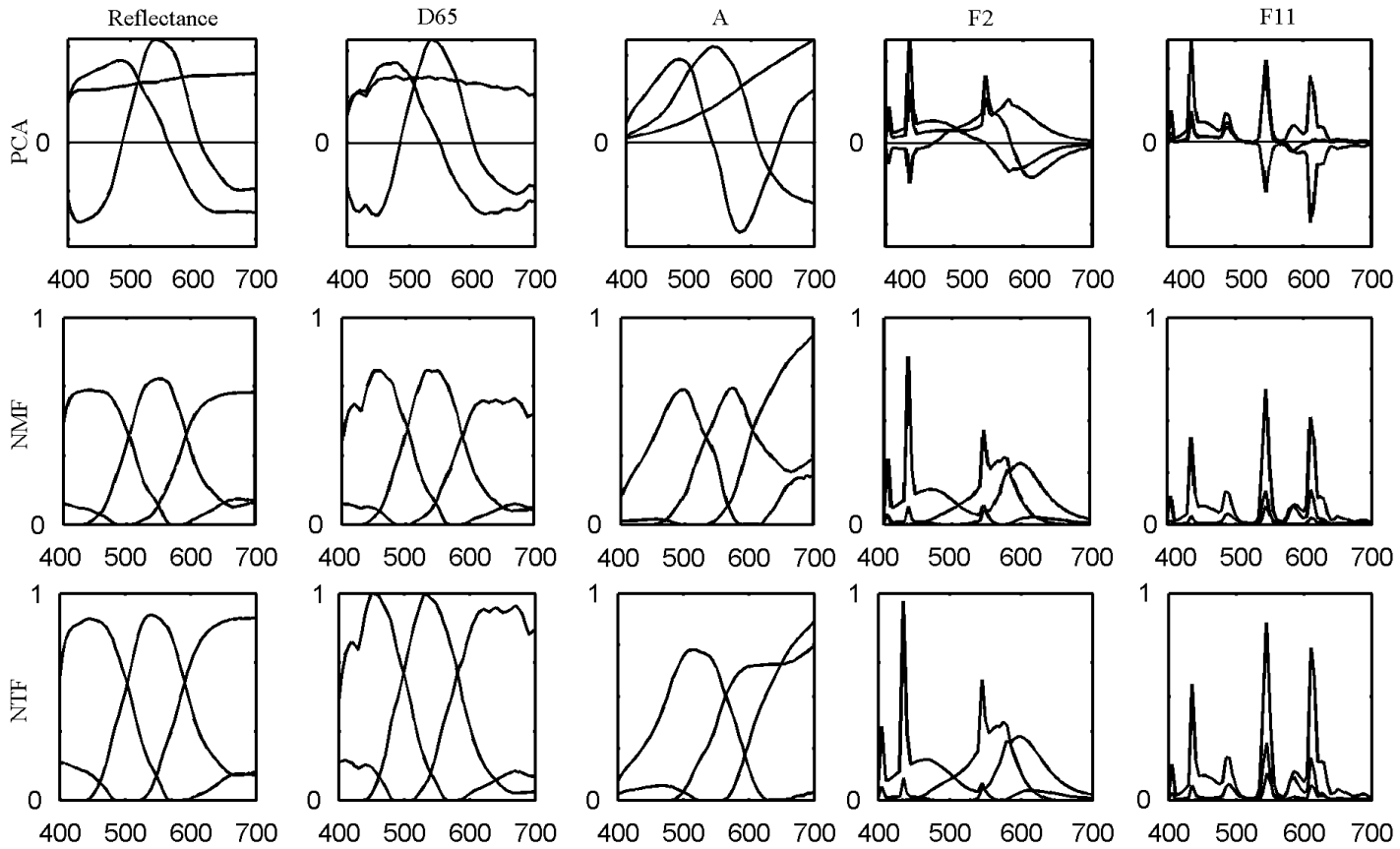


- Structure of the performed experiments:

- Tested spectra (Natural):



Results of the experiments



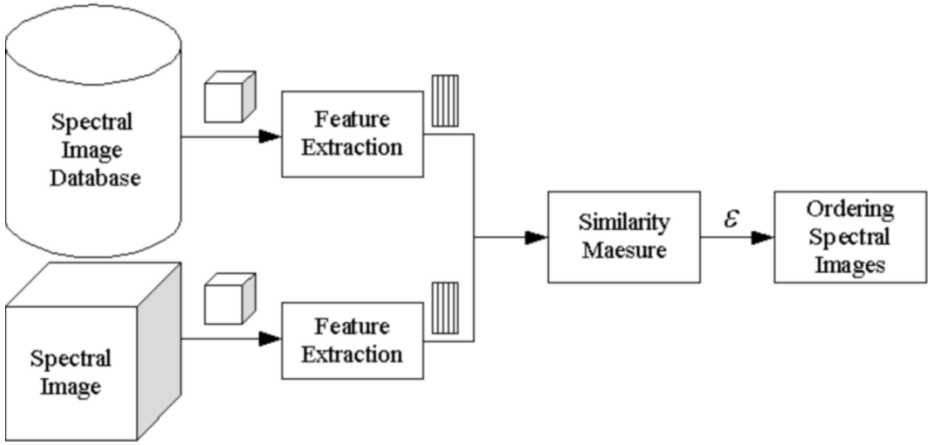
Reconstruction bases for Natural set for PCA, NMF, and PCA for used light sources.

Conclusion

The obtained results show that the reconstruction bases are strongly affected by light source. The properties of the applied lights surfaces are clearly reflected in the corresponding bases. Smooth lights give smooth bases. Peaks arise in the corresponding bases in a same order.

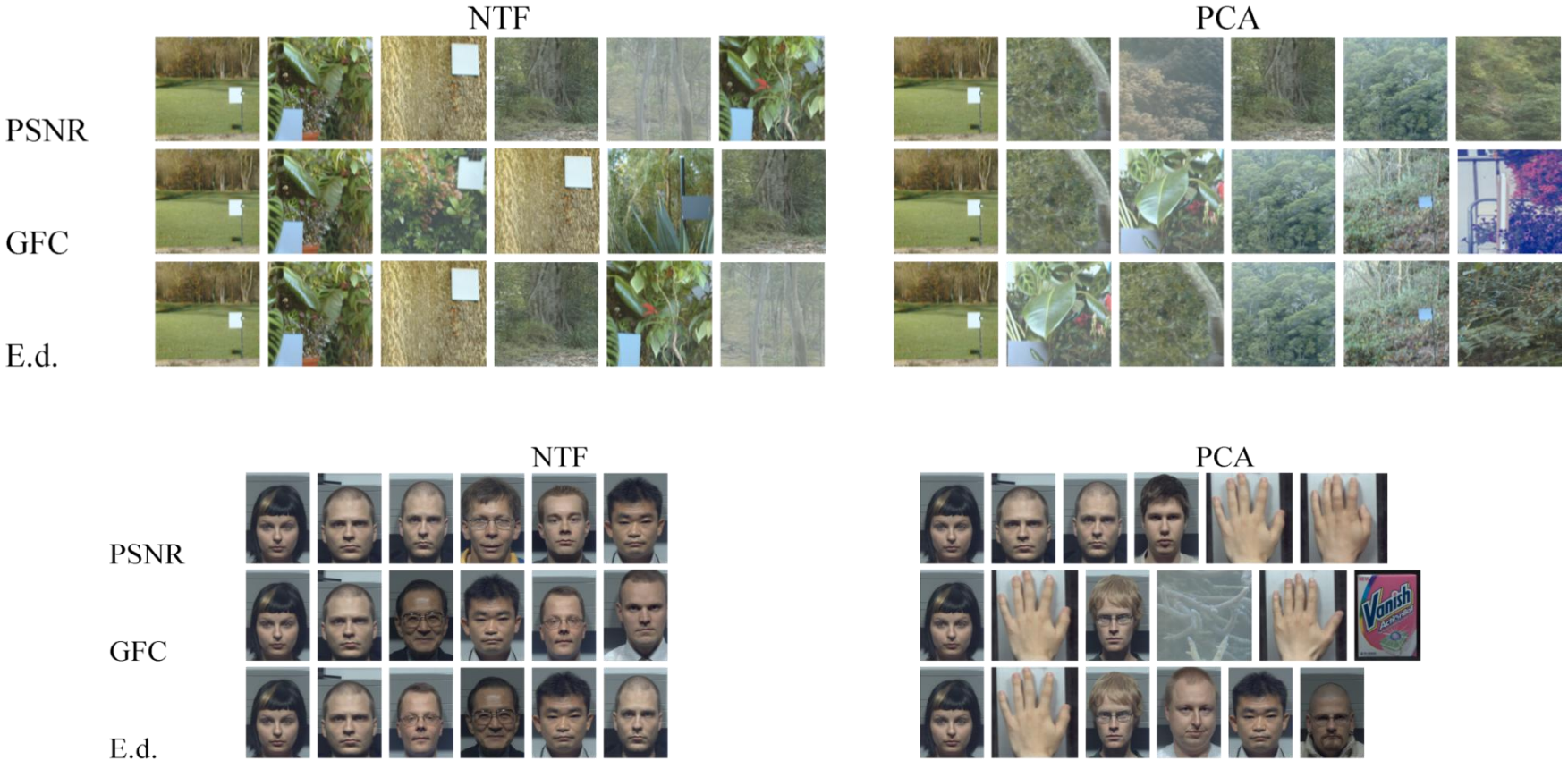
Searching Method

- NTF and PCA take spectra domain basis from each spectral image. Those basis defines as spectral color features.
- Similarity measures (Euclidian distance, goodness of fit coefficient (GFC) and peak signal-to-noise ratio (PSNR)) are calculated between spectra color features.



Experiments

Results



Results