PCA

Principal Component Analysis

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- Theoretically color spectrum is a
  - continuous
  - square integrable
  - in closed interval $[\lambda_1,\lambda_2]$ defined function

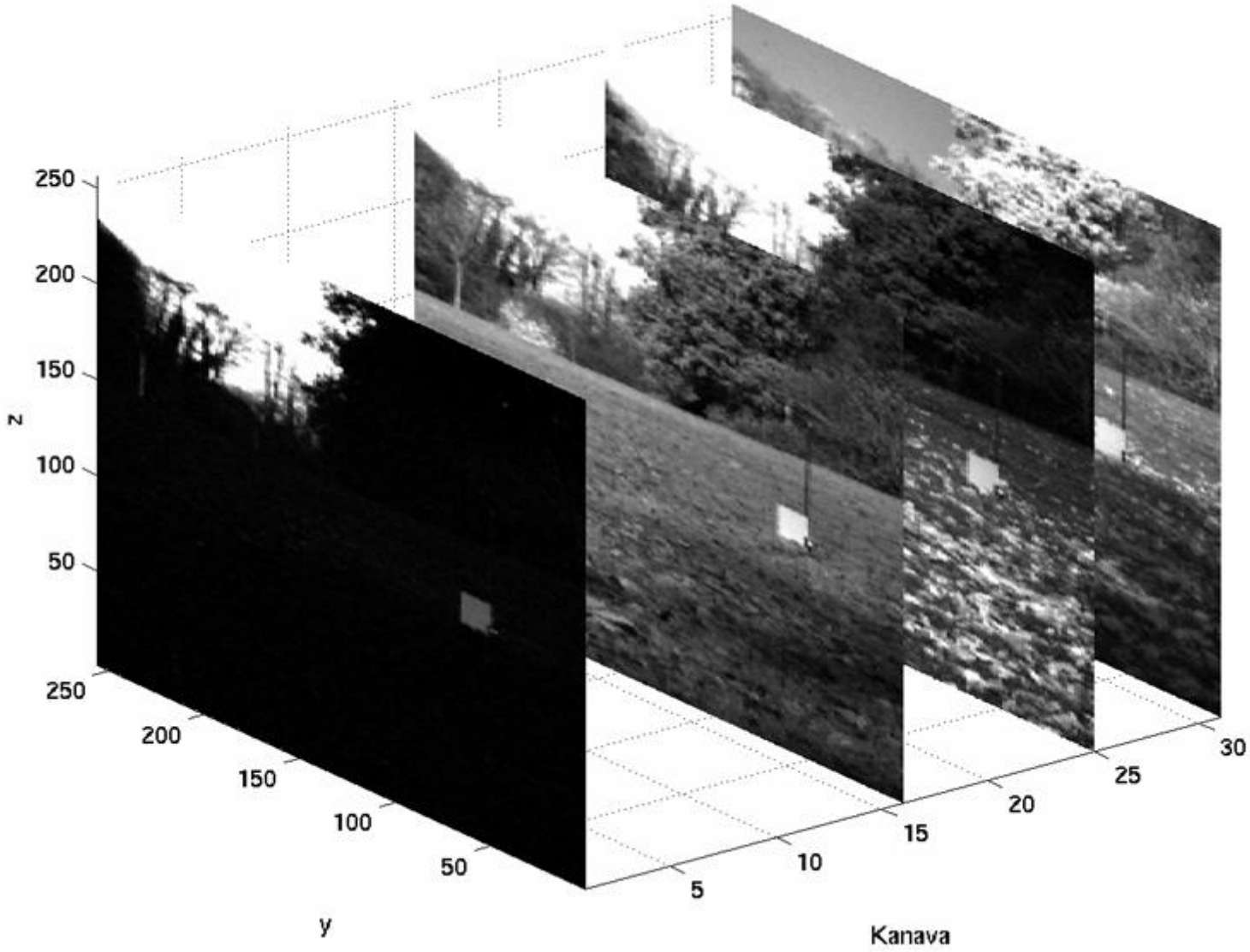
$$s(\lambda)$$

- In practice, it is an element of a n-dimensional vector space

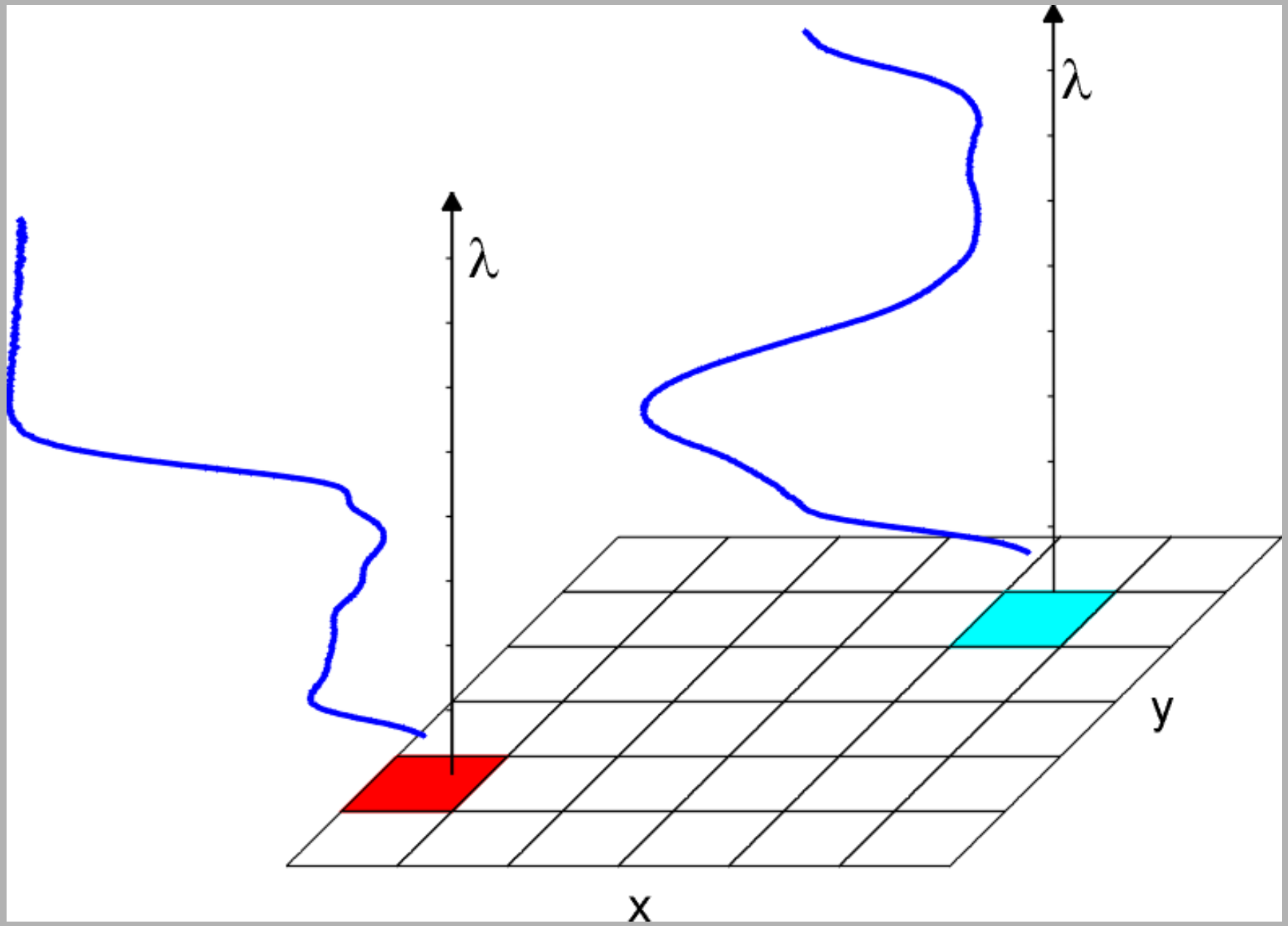
$$s(\lambda_n)$$

- n may range from 10 to >300

Components of a spectral image



Pixels of a spectral image



Color Filter Design

- One approach: to choose an optimized set of commercially available color filters (for example, interference filters, Kodak Wratten gelatin filters)

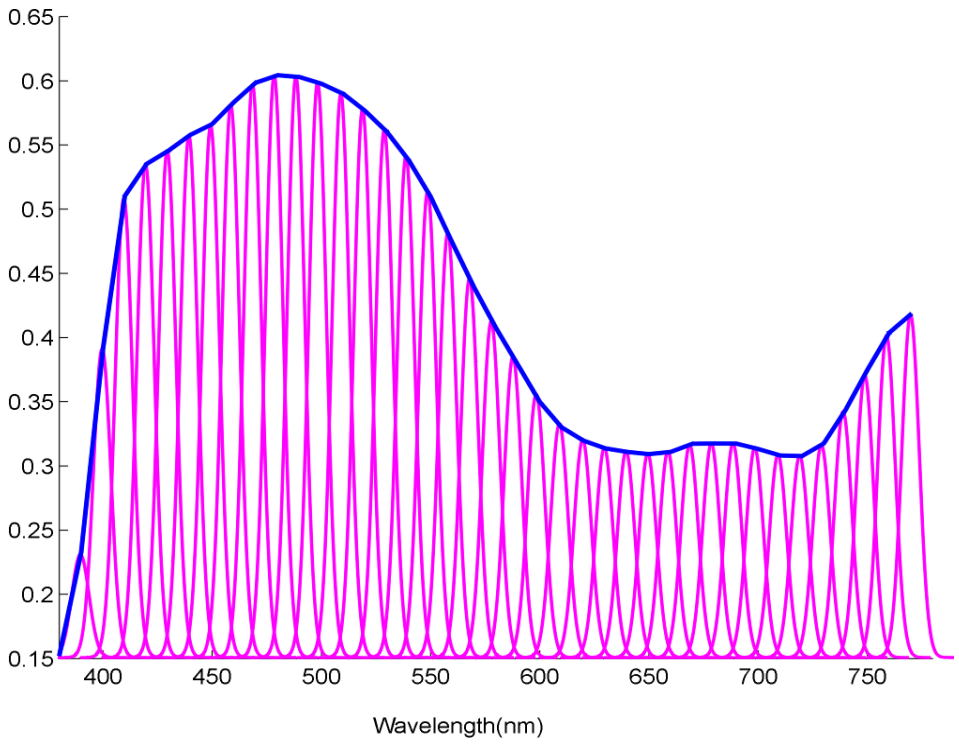
- Other approach: to design optimal color filters computationally

$\Rightarrow$ adaptive to various application

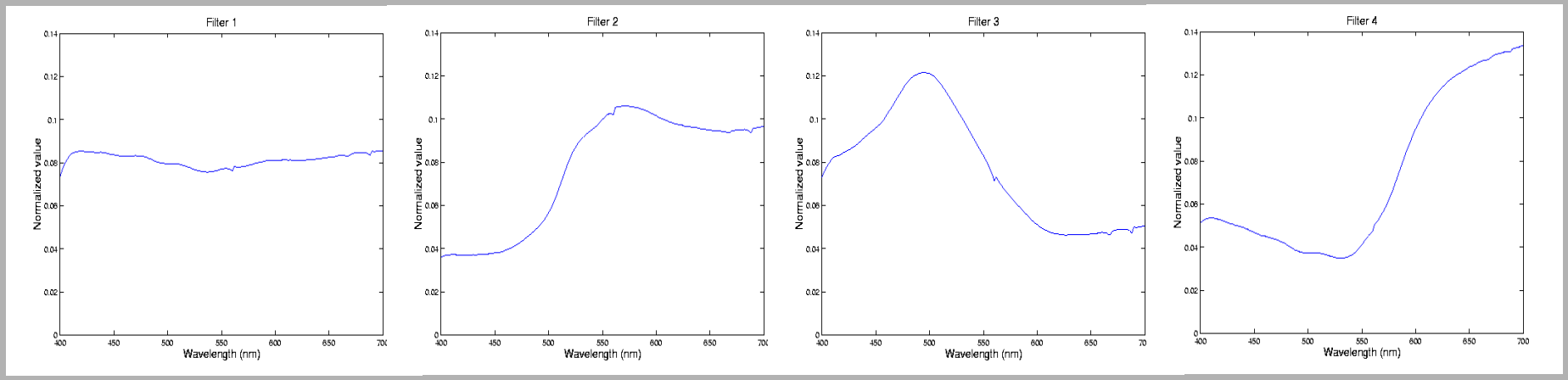
$\Rightarrow$ rewritable filter based imaging system

$\Rightarrow$ Spectral image can be reconstructed computationally, if needed

A spectrum sampled at 39 wavelengths



Designed color filters



Optically acquired inner-product images

FILTER 1

FILTER 2

FILTER 3

FILTER 4

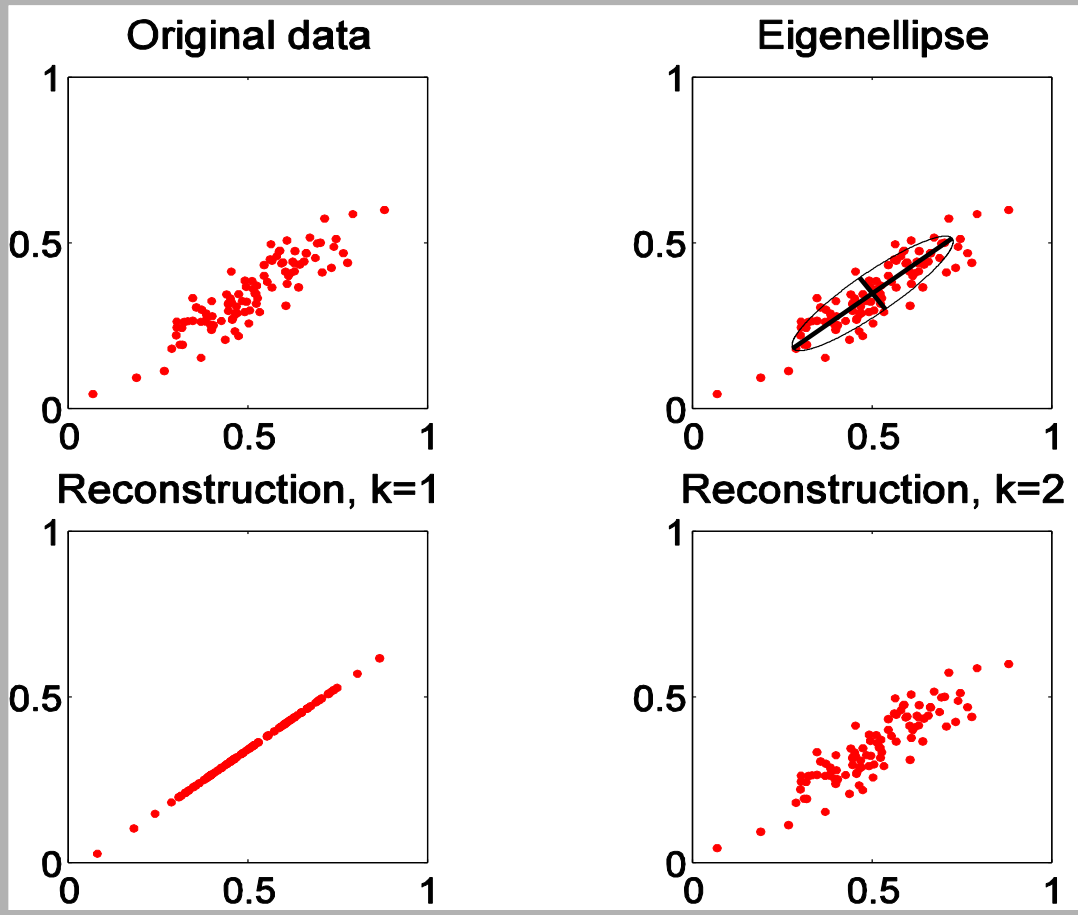
Algorithm PCA

1. Calculate correlation matrix for the data set
2. Calculate eigenvalues and eigenvectors for the correlation matrix
3. Select eigenvectors corresponding to the largest eigenvalues as a new basis for the data set
4. Calculate principal components between eigenvectors and the data set

- Applications: compression, pattern recognition etc. Frequently used for spectral image data

Standard PCA. Example

- The first principal component is used when *k=1*
- The first and second principal components are used when *k=2*. Reconstruction is perfect



Spectral Images & Principal Component Analysis (PCA)

spectrum

$$\mathbf{s}(\lambda) = (s(\lambda_1)\ s(\lambda_2)\ \ldots\ s(\lambda_n))^T$$

2-D spectral image

$$\mathbf{S} = \begin{pmatrix} s_1(\lambda_1) & \cdots & s_m(\lambda_1) \\ \vdots & \ddots & \vdots \\ s_1(\lambda_n) & \cdots & s_m(\lambda_n) \end{pmatrix}$$

1. calculating correlation matrix and eigenvectors, selecting base vectors

$$\mathbf{R} = \frac{1}{m}\mathbf{S}\mathbf{S}^T \qquad \mathbf{R}\Phi = \sigma\Phi$$

$$\mathbf{B} = \begin{pmatrix} b_1(\lambda_1) & \cdots & b_1(\lambda_n) \\ \vdots & \ddots & \vdots \\ b_m(\lambda_1) & \cdots & b_m(\lambda_n) \end{pmatrix}$$

Matrix B needs to be transposed

2. calculating inner product images

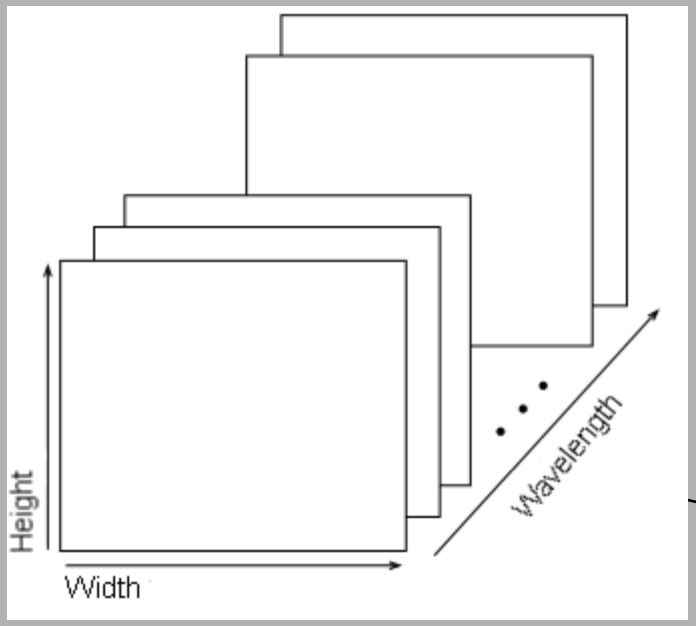
$$\mathbf{P} = \mathbf{B}^T\mathbf{S}$$

3. reconstructing spectral image

$$\tilde{\mathbf{S}} = \mathbf{B}\mathbf{B}^T\mathbf{S} = \mathbf{B}\mathbf{P}$$

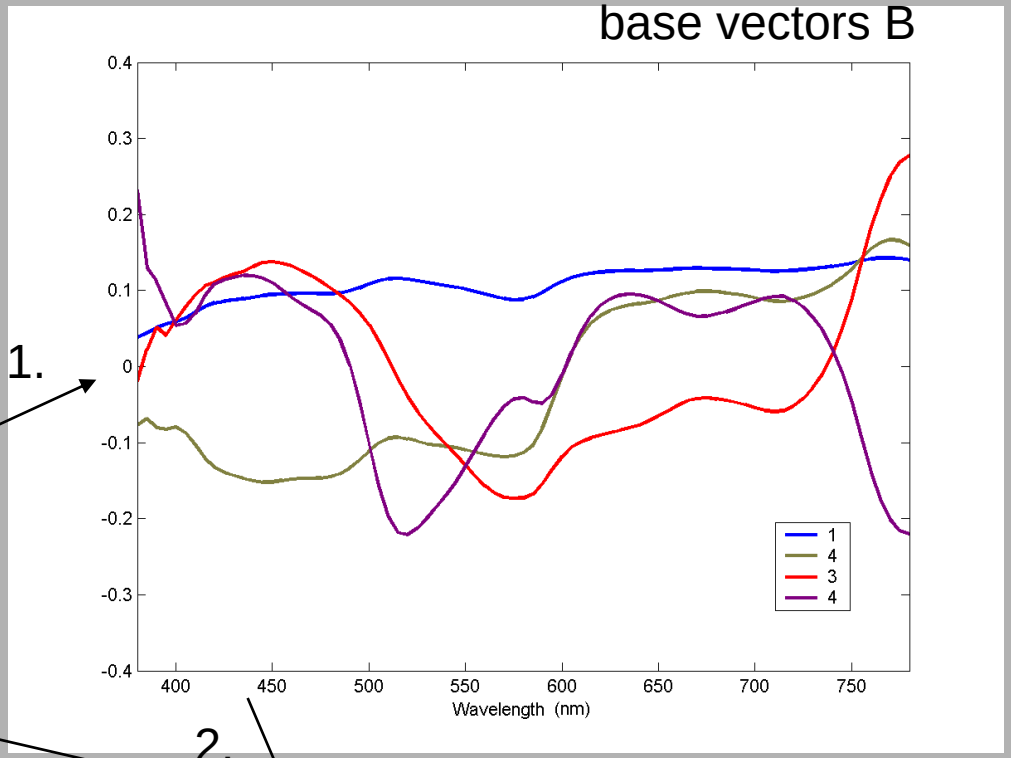
Illustration of PCA

original spectral image S



1.

base vectors B



2.

inner product images P

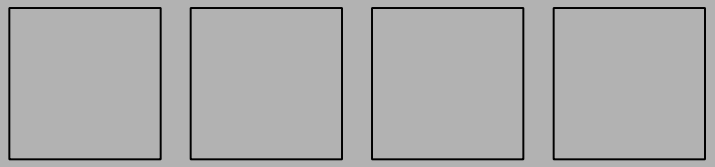
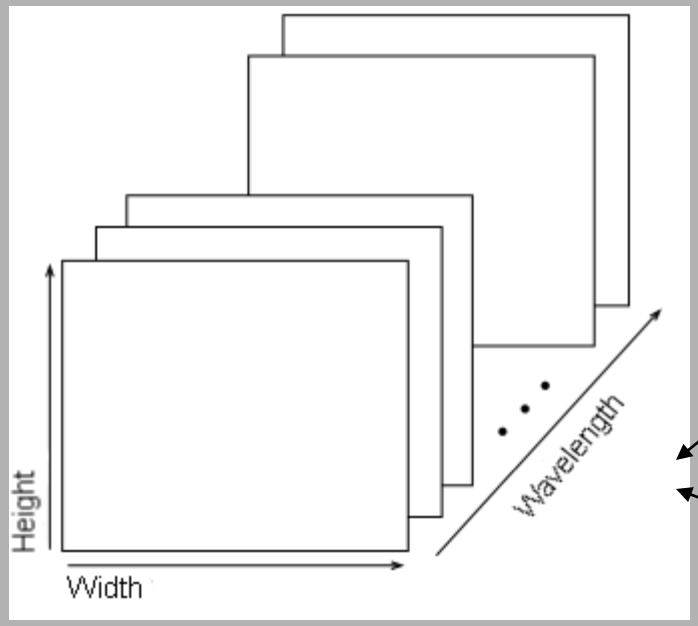
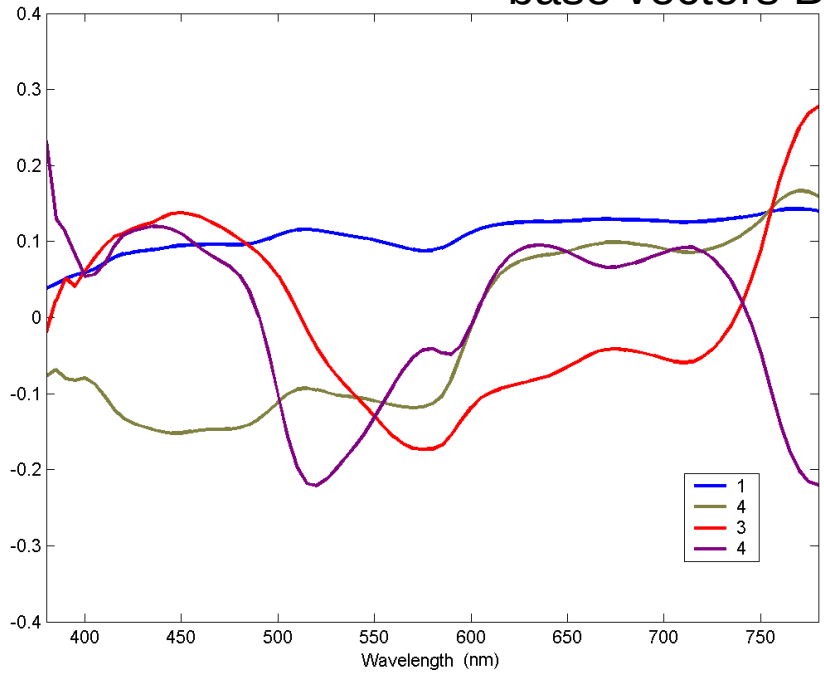


Illustration of PCA

reconstructed spectral image S

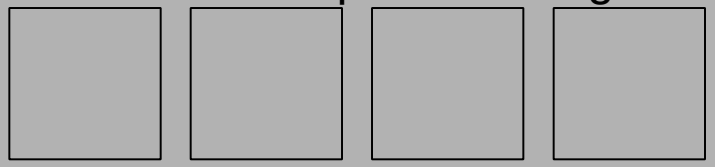


base vectors B

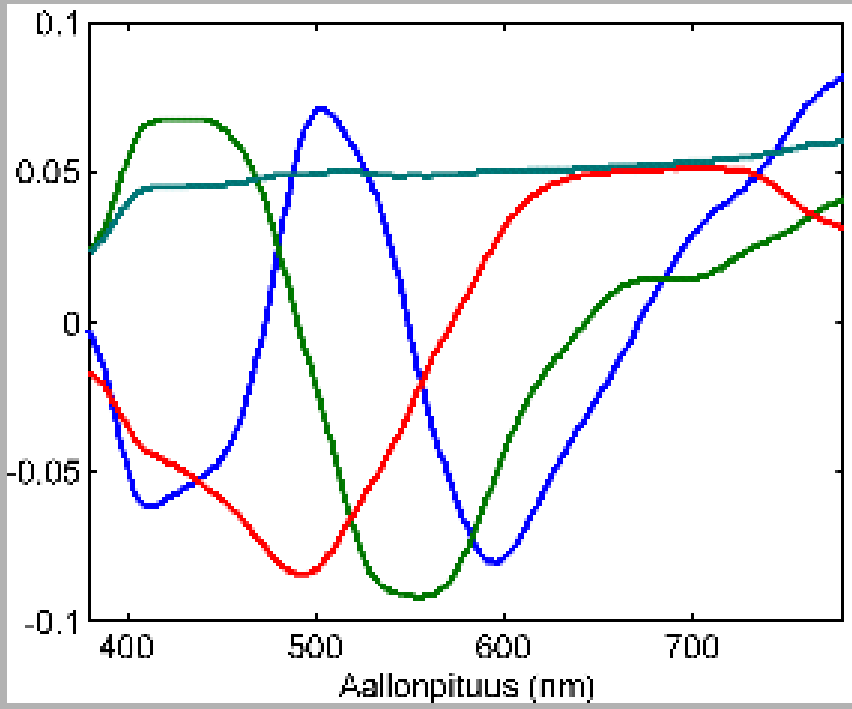


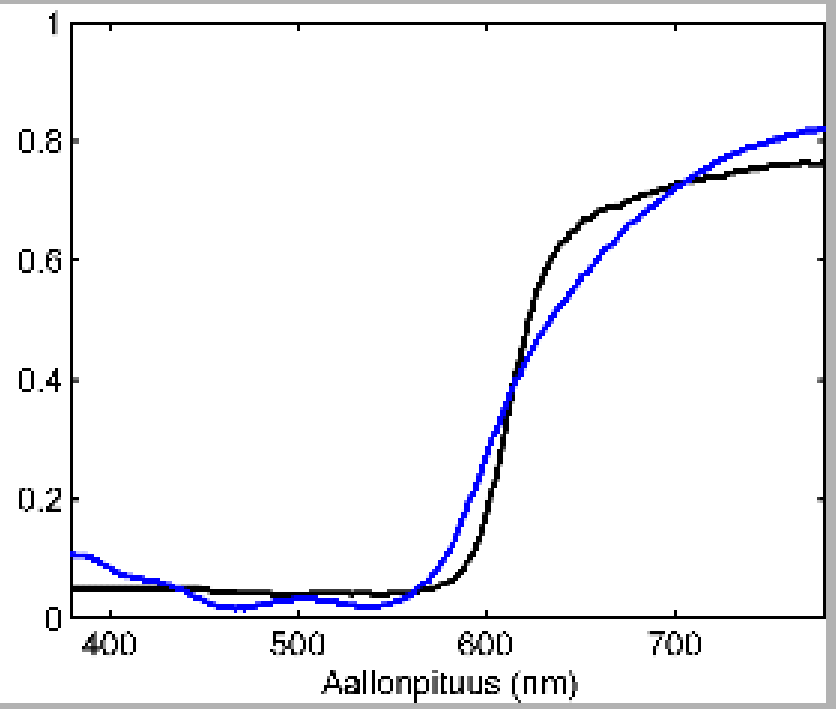
3.

inner product images P



Reconstruction of a spectrum

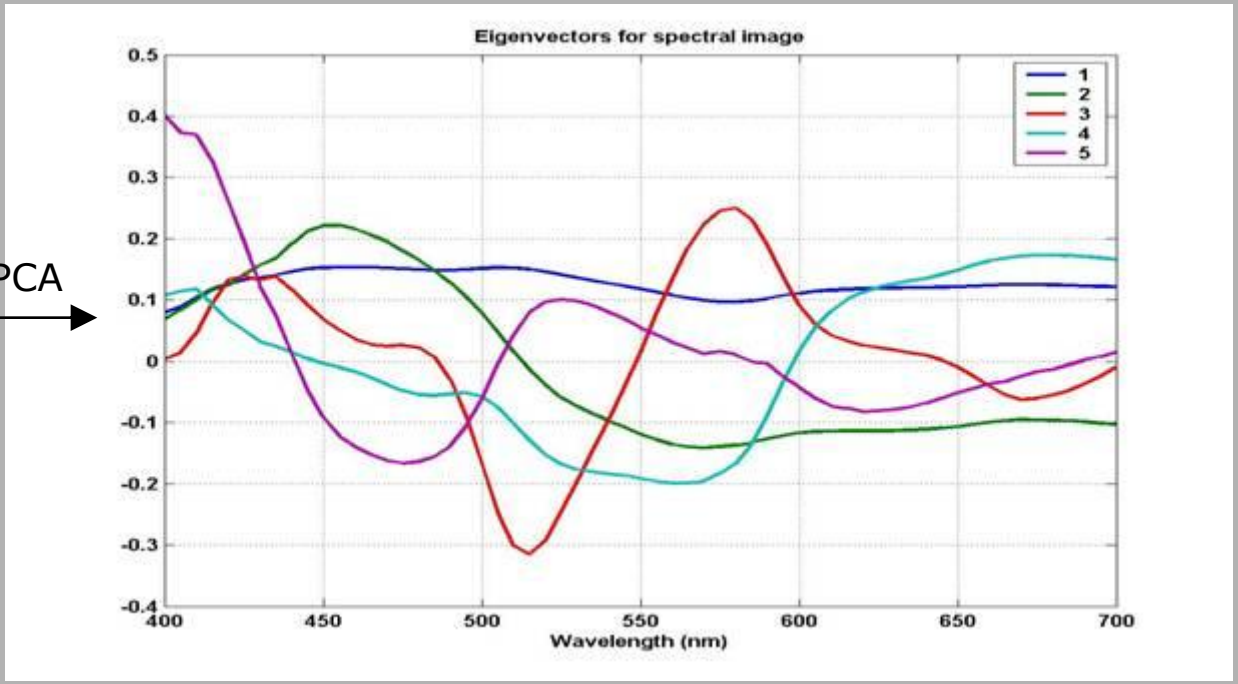




Component images of a spectral image

Spectral image as RGB-image

PCA



Inner-product images between the spectral image and eigenvectors:

1

2

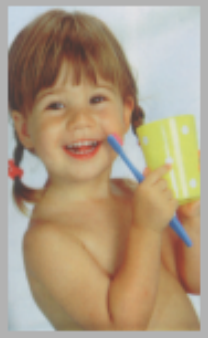
3

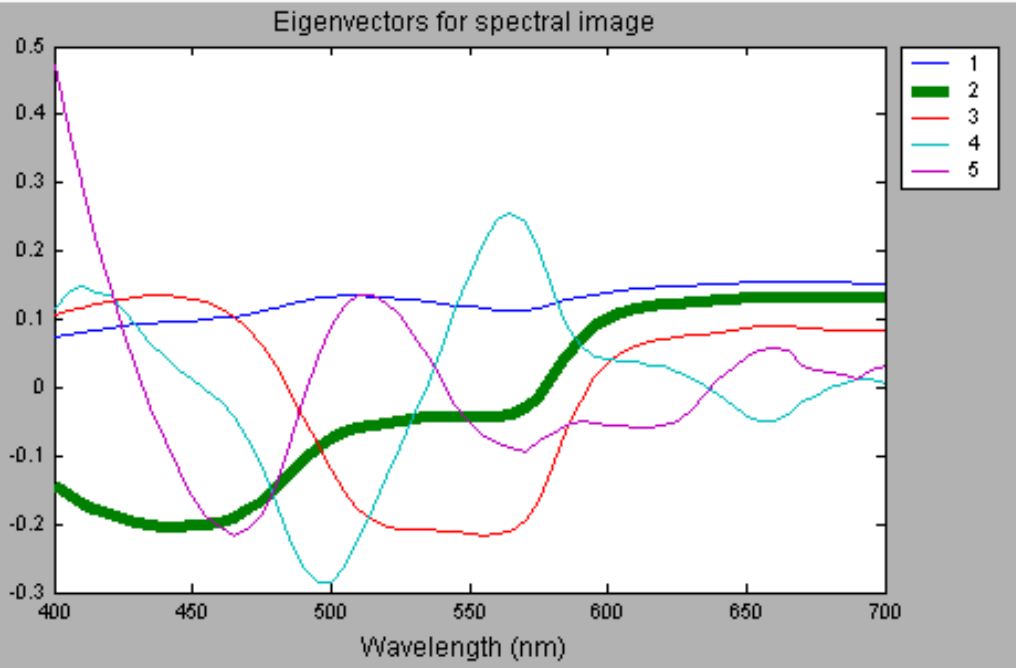
4

5

Principal Component Analysis

File Help





Load image

Wavelength
400:5:700

Change wavelength

Number of eigenvectors: 5

Update view

Select images for reconstruction

Reset selections

Fidelity 99.9937 %

Inner product images between spectral image and eigenvectors:

Images selected for reconstruction: All

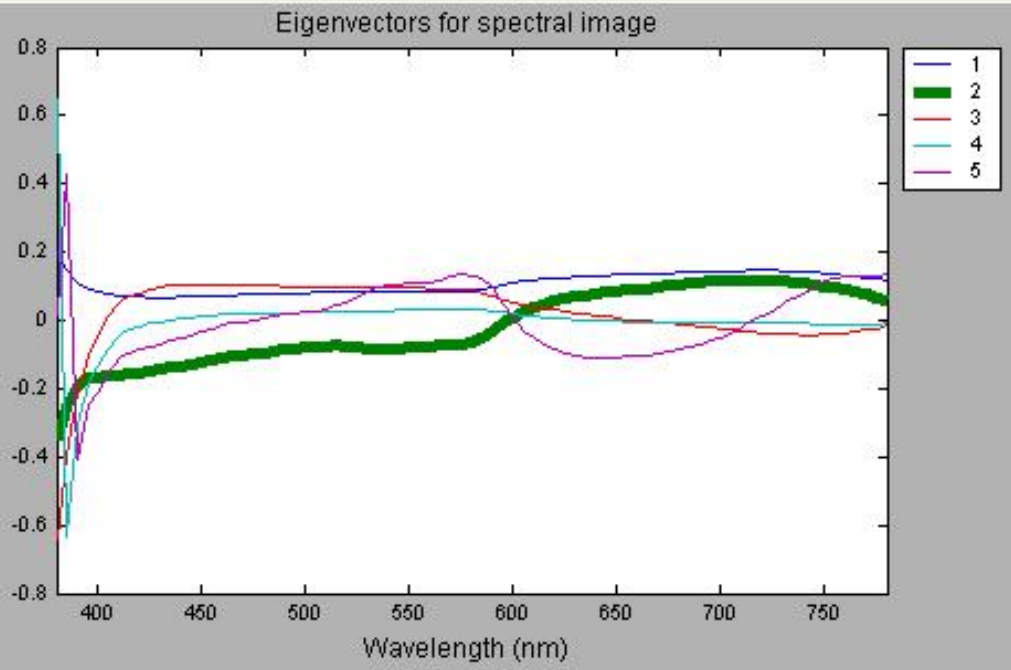
1 2 3 4 5

Inner product images

Compression errors

Principal Component Analysis

File Help



Load image

Wavelength
380:5:780

Change wavelength

Number of eigenvectors: 5 Update view

Select images for reconstruction Reset selections

Fidelity 99.8986 %

Inner product images between spectral image and eigenvectors:

Images selected for reconstruction: All

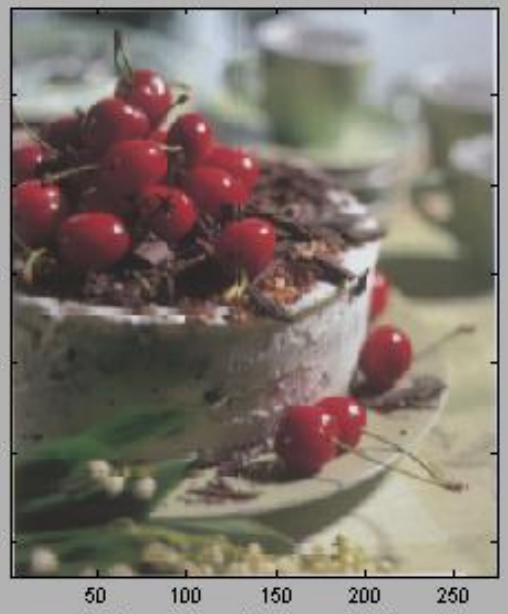
1 2 3 4 5

Inner product images Compression errors

Principal Component Analysis

File Help

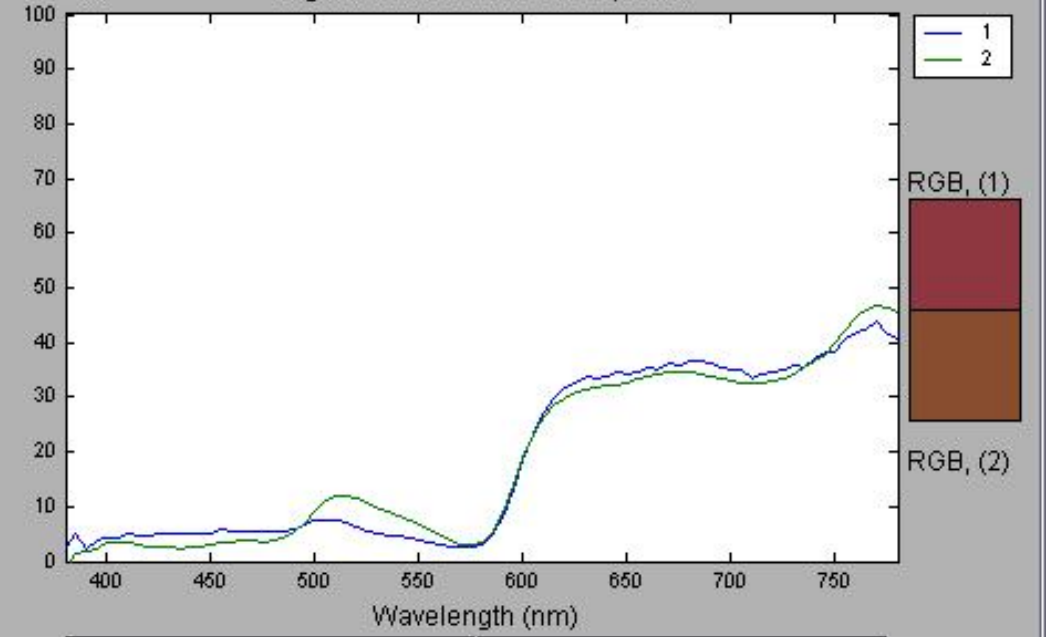
Original image (1)



Reconstructed image (2)

Inner product images | Compression errors

Original and reconstructed spectra



Compression Rate	26.98	pixel 97,111 :	
Average errors:		Errors of selected spectrum:	
Δ E	7.54	Δ E	21.5
Δ E S-CIELAB	6.35	Δ E S-CIELAB	5.34
Peak Signal-to-Noise Ratio	34.3	Peak Signal-to-Noise Ratio	32.42
Mean Square Error	0.00037	Mean Square Error	0.00057

Zoom image | Original size | Actual pixels

Select light source: A, C, D50, D65

Select system: 1931, 1964

Spectrum type: Reflectance, Relative radiance

Select other source

Spectral color analysis